Design of a knowledge-based force field for off-lattice simulations of protein structure**

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Prediction of protein structure from amino-acid sequence still continues to be an unsolved problem of theoretical molecular biology. One approach to solve it is to construct an appropriate (free) energy function that recognizes the native structures of some selected proteins (whose native structures are known) as the ones distinctively lowest in (free) energy and then to carry out a search of the lowest-energy structure of a new protein. In order to reduce the complexity of the problem and the cost of energy evaluation, the so-called united-residue representation of the polypeptide chain is often applied, in which each amino-acid residue is represented by only a few interaction sites. Once the global energy minimum of the simplified chain has been found, the all-atom structure can easily and reliably be constructed. The search of the lowest-energy structure is usually carried out by means of Monte Carlo methods, though use of more efficient global-optimization methods, especially those of deformation of original energy surface is potentially promising. Monte Carlo search of the conformational space can be accelerated greatly, if the chain is superposed on a discrete lattice (the on-lattice approach). On the other hand, the on-lattice approach prohibits the use of many efficient global-optimization methods, because they require both energy and its space derivatives. The on-lattice methods in which the chain is embedded in the continuous 3D space are, therefore, also worth developing. In this paper we summarize the work on the design and implementation of an off-lattice united-residue force field that is underway in our group, in cooperation with Professor H.A. Scheraga of Cornell University, U.S.A.

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Abbreviations: APP, avian pancreatic polypeptide; DEM, diffusion equation method; EDMC, electrostatically driven Monte Carlo method; MCM, Monte Carlo method with minimization; p, united peptide group; PDB, Protein Data Bank; r.m.s., root mean square; SC, united site chain; SCMTF, self consistent mean torsional field.
Proteins are an essential constituent of all known living organisms. Fibrous proteins, such as collagen or keratin, are main tissue building stuff, while globular proteins provide the necessary micro-environment for the chemical reactions of the cellular system (enzymes), serve as information relayers to and from the nervous system (neurotransmitters), or disclose intruder microorganisms and other alien potentially malignant bodies (the proteins of the immune system). All these diverse functions are associated with the ability of a protein to maintain its unique three-dimensional structure, the so-called native structure, under physiological conditions. This structure is, in turn, unequivocally determined by the amino-acid sequence. The knowledge of the native structure is a necessary condition to learn about the physiological role and the mechanism of the action of a protein.

X-ray crystallography, NMR spectroscopy, and other experimental methods of structure solving provide only about several hundred new structures a year, while in the same period of time ten thousands of new amino-acid sequences are revealed. The design of a reliable method of protein-structure prediction is therefore of vital importance. There are essentially two classes of approaches to this problem: the homology methods and the methods based on energetic criteria. In the first case, the unknown structure is constructed based on known structural motifs whose amino-acid sequences are similar to the sequence studied, taking advantage of empirical relationship between sequence and the 3D structure [1–5]. The methods of the second group are based on the thermodynamic hypothesis formulated by Anfinsen and co-workers [6], according to which the native structure of a protein is the global minimum of its free energy under given conditions.

An early criticism of the thermodynamic hypothesis was based on the fact that the time required to carry out a systematic search of the conformational space of even a small protein would be comparable to the age of the Earth, while in reality proteins fold in seconds or in minutes. This time-scale discrepancy is known as the Levinthal paradox [7]. Studies towards solving the Levinthal paradox were undertaken by Sali and co-workers [8], Wolynes and coworkers [9], as well as Hao & Scheraga [10, 11]. Based on simple polypeptide-chain and interaction-scheme models, these authors found that if the native structure is separated from the non-native structures by a sufficiently large energy barrier, the simulated folding process is very fast. Otherwise, the lowest-energy structure might never be reached. This behavior can be explained by substantial narrowing down of the number of conformational states with lowering the energy in the first case. Thus, after low-energy structures have been reached, the system needs to sample only a comparatively small number of states. If, however, the energy spectrum is quasi-continuous in the whole range, the number of conformational states does not narrow down significantly upon energy lowering and the system becomes frustrated. In view of this, Anfinsen’s thermodynamic hypothesis should be formulated more rigorously: the native structure not only is the global minimum of protein’s free-energy, but is also separated by a sufficiently large energy barrier from the non-native structures.* In conclusion, there are foldable and non-foldable amino-acid sequences and the former were chosen during the evolution as components of living organisms.

In order to make use of the thermodynamic hypothesis, reliable energy functions and efficient global-optimization methods are required to reproduce protein free energy surface and search its conformational space, respectively. These problems are strongly interrelated, because only with an efficient global-optimization method assessment can

*It should be kept in mind that because a protein molecule is subject to continuous fluctuations, the native structure should be regarded as a family of conformations very similar in geometry, rather than as a single well-defined conformation. Thus, the term global minimum of protein’s energy surface should be regarded as an abbreviation for the “deepest depression” in energy surface that contains many small pits corresponding to interrelated conformations constituting the native structure.
be made, whether an energy function is a folding potential for an amino-acid sequence known experimentally as foldable and whether it leads to the experimentally known native structure.

Despite the progress made in the recent years in the design of the global-optimization methods [12–24], it is still beyond reach to search the global energy minimum of a protein at the all-atom resolution. Therefore, simplified representations of the polypeptide chain, in which each amino-acid residue is modeled by a few interaction sites, each of which comprises several real atoms (the so-called united-residue representations*) receive great attention, since the pioneering works of Levitt & Warshell [25] and Levitt [26]. Following these works, a considerable number of united residue force fields were designed [9, 27–63]. After the global energy minimum has been found for the simplified chain, it can be converted to the all-atom chain, and limited exploration of the conformational space of the all-atom chain can then be carried out in order to locate the global minimum in the all-atom representation [38, 39, 41, 59, 60]. Such a protocol has recently been developed and implemented with considerable success by Koliński, Skolnick and co-workers [38, 39, 41] in predicting the three-dimensional structures of model monomeric helical proteins crambin (which also contains a β-sheet section) [41], and the dimeric GCN4 leucine zipper [42, 45] and Liwo et al. [59, 60], who succeeded in predicting the three-dimensional structure of the avian pancreatic polypeptide.

There are two ways to explore the conformational space of polypeptide chains with the use of a united-residue potential: the on-lattice and the off-lattice approach. In the first case, the polypeptide chain is superposed on a discrete lattice, and the number of possible conformations is, therefore, finite. In the simplest approach, the interaction potential is reduced to a set of residue-residue contact free energies [30–34, 52–54]. The rationale for such an approach was based on the assumption that side-chain packing is the principal driving force in protein folding [64]; more recent studies, however, have shown that this assumption is probably not true [36]. The recent approach developed in Skolnick's group incorporates many different interactions that can be responsible for protein folding [64]: side-chain packing, local interactions, hydrogen bonding, surface energy, and cooperativity in side-chain packing and hydrogen bonding [37–40]. The parameters of the potentials for on-lattice simulations were determined from a statistical analysis of the distributions of interacting sites obtained from the crystal data of known proteins collected in the Brookhaven Protein Data Bank (PDB) [65].

Work on the off-lattice united-residue potentials was initiated even earlier than on the on-lattice ones [25–29, 47–51, 54–63]. These potentials have also been used with considerable success to predict the three-dimensional structure of known proteins [50, 51, 56–58, 60, 62]. In contrast to the on-lattice potentials, they are functions of continuous variables. Therefore, the off-lattice approach to protein folding enables using many powerful techniques for global-search minimization that require not only the energy, but also its spatial derivatives, e.g. Monte Carlo with Minimization (MCM) [16, 17], the Diffusion Equation Method (DEM) [19, 20], the Self Consistent Mean Torsional Field (SCMTF) [21], or the shift method [24]. In the last years we have undertaken the work on the design of a united-residue potential of this class and the associated procedure of the prediction of protein structure from sequence. This work is summarized in the present article.

The rest of this paper organized as follows. First, the united-residue representation de-

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*As pointed out by one of the referees, the term united residue could misleadingly suggest that each amino-acid residue is represented by one interaction site, which is not always the case, and therefore the term united atom could be more appropriate. However, the literature usage of the latter term always means a non-hydrogen atom fused with the attached hydrogens, while the term united residue is used specifically to denote coarse-grained models of polypeptide chains. Therefore, in order to maintain consistency with the literature, we keep the wording united residue throughout this paper.
developed in our group and the associated energy function are described and discussed. Then, the procedure of parameterization of the force field is described. Next, method used for the conversion of the simplified chain into an all-atom chain is presented. Finally, results of the application of the force field and whole procedure are described, including the inverse- and de novo folding tests.

REPRESENTATION OF POLYPEPTIDE CHAINS AND INTERACTION SCHEME

In our model [60, 61], a polypeptide chain is represented by a sequence of \( \alpha \)-carbon (\( C^\alpha \)) atoms linked by virtual bonds with attached united side chains (SC) and united peptide groups (p) located in the middle between the consecutive \( \alpha \)-carbons. Only the united peptide groups and united side chains serve as interaction sites, the \( \alpha \)-carbons assisting in the definition of the geometry (Fig. 1). All the virtual bond lengths (i.e. \( C^\alpha-C^\alpha \) and \( C^\alpha-SC \)) are fixed; the \( C^\alpha-C^\alpha \) distance is taken as 3.8 Å which corresponds to trans peptide groups. In the current version of the force field, we allow, however, for variation of the side-chain positions with respect to the backbone (\( \alpha_{SC} \) and \( \beta_{SC} \)), and for the variation of the virtual-bond angles \( \theta \); in our earlier approach [59, 60] they were assumed fixed at the value of 90° (the most probable value as found by the analysis of the PDB [66]) and the average side-chain geometry relative to the three adjacent \( C^\alpha \)s, as found by Levitt [26].

The energy of the virtual-bond chain is expressed by Eqn. (1).

\[
U = \sum_{i<k} U_{SC_i SC_k} + \sum_{i\neq j} U_{SC_i p_j} + \\
+ \omega_{el} \sum_{i<j-1} U_{el} + \omega_{tor} \sum_{i} U_{tor}(\gamma_i) + \\
+ \omega_{rot} \sum_{i} \left[ U_{rot}(\theta_i) + U_{rot}(\alpha_{SC_i}, \beta_{SC_i}) \right] + \\
+ \omega_{corr} U_{corr}
\]  

(1)

where \( U_{SC_i SC_j} \), \( U_{SC_i p_j} \), and \( U_{p,p_j} \) denote the energies of the interactions between side

![Figure 1. United-residue representation of a polypeptide chain.](image)

The interaction sites are side-chain (SC) centroids of different sizes and peptide-bond centers (p) indicated by dashed circles, while the \( \alpha \)-carbon atoms (small empty circles) are introduced only to assist in defining the geometry. The virtual \( C^\alpha-C^\alpha \) bonds have a fixed length of 3.8 Å, corresponding to a trans peptide group; the virtual-bond (\( \theta \)) and dihedral (\( \gamma \)) angles are variable. Each side chain is attached to the corresponding \( \alpha \)-carbon with a fixed "bond length", \( b_{SC_i} \), variable "bond angle", \( \alpha_{SC_i} \), formed by SC and the bisector of the angle defined by \( C^\alpha_{i-1}, C^\alpha_i, \) and \( C^\alpha_{i+1} \), and with a variable "dihedral angle" \( \beta_{SC_i} \) of counterclockwise rotation about the bisector, starting from the right side of the \( C^\alpha_{i-1}, C^\alpha_i, C^\alpha_{i+1} \) frame.
chains, between side chains and peptide groups, and between peptide groups, respectively, $U_{\text{rot}}(\gamma_i)$ denotes the energy of variation of the virtual-bond dihedral angle $\gamma_i$, $U_{\text{rot}}(\theta_i)$ denotes the "bending" energy of the virtual-bond angle $\theta_i$, $U_{\text{rot}}(\alpha_{SC_i}, \beta_{SC_i})$ is the local energy of side chain $i$, $U_{\text{corr}}$ includes cooperative terms (e.g. the four body interactions considered by Koliński, Skolnick and coworkers [38]), and the $o$'s denote relative weights of the respective energy terms.

The term $U_{SC_i, SC_j}$ comprises the mean free energy of the hydrophobic (hydrophilic) interactions between the side chains. It therefore contains implicitly the contributions coming from the interactions with the solvent. Its functional form is expressed by Eqn. (2).

$$U_{ij} = 4\left[\xi_{ij} x_{ij}^{12} - \xi_{ij} x_{ij}^6\right]$$

where $\xi_{ij}$ is the pair-specific van der Waals well-depth; $\xi > 0$ corresponds to hydrophobic-hydrophobic-type and $\xi < 0$ to hydrophobic-hydrophilic and hydrophilic-hydrophilic-type interactions (see Fig. 2 for illustration). The quantity $x_{ij}$ is the reciprocal of the reduced distance between side chains; it can depend on their distance alone for radial-only potential (in this case $x_{ij} = \sigma_{ij}^o / r_{ij}$, $r_{ij}$ being the distance between the side chains and $\sigma_{ij}^o$ a pair-specific constant that depends on the types of side chains $i$ and $j$) or on both distance and orientation; the same applies to $\xi_{ij}$. In our work we have considered and parameterized functional form of both types. The functional forms of $\xi$ and $x$ can be found in the original paper [61].

The peptide-group interaction potential ($U_{p,p}$) accounts mainly for the electrostatic interactions between them or, in other words, for their tendency to form backbone hydrogen bonds. In contrast to $U_{SC_i, SC_j}$, its functional form was derived rigorously by averaging the simplified electrostatic-interaction energy of the peptide groups over the angles $\lambda$ of their rotation about the corresponding $C^\alpha-C^\alpha$ virtual bond axes, assuming that each peptide group is modeled by a point dipole located in the middle of the virtual bond, as was proposed by Piela & Scheraga [15] (Fig. 3). The potential is expressed by Eqn. (3); the details of the derivation and parameterization can be found in the papers cited [59, 60].

$$U_{p,p} = \frac{A_{p,p}}{r_{ij}^3} \left[\cos \alpha_{ij} - \frac{3 \cos \beta_{ij} \cos \gamma_{ij}}{r_{ij}} - \frac{B_{p,p}}{r_{ij}^6} \left[4 + \cos \alpha_{ij} - \frac{3 \cos \beta_{ij} \cos \gamma_{ij}}{r_{ij}} - \frac{3(\cos^2 \beta_{ij} + \cos^2 \gamma_{ij})}{r_{ij}} + \frac{\epsilon_{p,p} \left(r_{ij}^o \ p_{i} p_{j} \right)^{12}}{r_{ij}^6} - \frac{2(\epsilon_{p,p} \ p_{i} p_{j})^6}{r_{ij}^6}\right]\right]$$

with

$$\cos \alpha_{ij} = v_i \cdot v_j$$
$$\cos \beta_{ij} = v_i \cdot e_{R_i}$$
$$\cos \gamma_{ij} = v_j \cdot e_{R_j}$$

where $A_{p,p}$, $B_{p,p}$, and $\epsilon_{p,p}$ are constants characteristic of the kind of interacting peptide groups, $r_{ij}$ is the distance between the peptide-group centers, $v_i$ is the unit vector pointing from $C^\alpha_i$ to $C^\alpha_{i+1}$, and $e_{R_i}$ is the unit vector pointing from $p_i$ to $p_j$ (see Fig. 3 for illustration). Two types of peptide groups were distinguished: ordinary and proline; the second one can act as hydrogen-bond acceptor only and also comprises all N-methylated amino-acid residues (e.g. sarcosine). This gives a total of three sets of constants in Eqn. (3). The angular part of Eqn. (3) favors parallel and near-parallel orientation of the virtual $C^\alpha-C^\alpha$ bonds, as
Figure 2. Plot of the energy of interaction between the hydrophobic ($\varepsilon > 0$) and hydrophilic ($\varepsilon < 0$) side chains (Eqn. 2) in their reduced distance.

Figure 3. The relative orientation of the virtual bonds $C_i^\alpha - C_{i+1}^\alpha$ and $C_j^\alpha - C_{j+1}^\alpha$ is described by the angles $\alpha_{ij}$, $\beta_{ij}$, and $\beta_{ij}$, defined by Eqn. (3).

The angle $\alpha_{ij}$ is not shown here because the two virtual bonds $C_i^\alpha - C_{i+1}^\alpha$ and $C_j^\alpha - C_{j+1}^\alpha$ are not necessarily coplanar. $\theta$ is the angle between two successive virtual bonds. The peptide-group dipole moments are represented by arrows (pointing from the carbonyl oxygen to the amide hydrogen of a peptide group), and the angles $\mu_i$ and $\mu_j$ between them and the virtual bonds are also shown, as well as the rotation angles $\lambda_i$ and $\lambda_j$ of the peptide-group dipoles. The two dipoles are separated by a distance $r_{ij}$. 
encountered in hydrogen-bonded backbone peptide groups.

The torsional energy, $U_{\text{tor}}$, is expressed in terms of a Fourier series in the virtual bond dihedral angles $\gamma$, as given by Eqn. (4).

$$
U_{\text{tor}} (\gamma) = \sum_{k=1}^{6} \left[ a_k (\cos k \gamma + 1) + b_k (\sin k \gamma + 1) \right]
$$

(4)

This energy reflects the local propensities of the polypeptide chain, i.e. to form the right rather than left-handed helices and the left rather than right-handed $\beta$-strands. It was natural to consider three torsional types of amino-acid residues: glycine (because of the absence of the $\beta$-carbon), proline (because of the restriction caused by the presence of the pyrrolidine ring), and alanine (which comprises all other amino-acid residues). Detailed analysis of local propensities of the structures contained in the PDB confirmed this division [62].

The expressions of the bending energy, $U_b$, and local side-chain energy, $U_{\text{rot}}$, have a form of the negative of the logarithms of sums of Gaussians; in the second case their centers correspond to different rotameric states of the side chains. Because these expressions are lengthy, the reader is referred to the original paper [62].

The multibody (or cooperative) term $U_{\text{corr}}$ arises from the fact that details of all-atom chain are lost when converting it into the simplified chain. Mathematically it can be expressed as averaging the energy over some "less important" degrees of freedom, as expressed by Eqn. (5) [63]:

$$
\mathcal{U}(\mathbf{x}) = \mathbb{E}^{-1} \left[ \frac{\int_{\mathcal{Y}} w(E(\mathbf{x}; \mathbf{y})) \frac{\partial E(\mathbf{x}; \mathbf{y})}{\partial E} dV_y}{\int_{\mathcal{Y}} w(E(\mathbf{x}; \mathbf{y})) dV_y} \right]
$$

(5)

where $\mathbb{E}$ is an unequivocal monotonic function in one variable, $\omega$ is a weight for the

energy, $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ and $\mathbf{y} = [y_1, y_2, \ldots, y_n]^T$ denote the "important" and the "less important" variable set, respectively.

The form of the averaged energy $\mathcal{U}$ will depend on the choice of the transformation $\mathbb{E}$ and the weight function $\omega$. Usually, the energy is Boltzmann-averaged [26, 60], which means that $\mathbb{E}(E) = E$ and $\omega(E) = \exp(-E/k_B T)$, $T$ being the absolute temperature and $k_B$ the Boltzmann constant. However, because the local and two-body terms can be identified with free energies rather than with Boltzmann-averaged energies [35–37, 56, 61, 62], it is more appropriate to choose $\mathbb{E}(E) = \exp(-E/k_B T)$ and $\omega(E) = 1$ or, alternatively, $\mathbb{E}(E) = \exp(E/k_B T)$ and $\omega(E) = \exp(-E/k_B T)$.

In this case, $\mathcal{U}(\mathbf{x})$ has the meaning of the free energy associated with a set of fixed values of the important variables $\mathbf{x}$, as expressed by Eqn. (6) [63].

$$
\mathcal{U}(\mathbf{x}) = F(\mathbf{x}) = -k_B T \cdot \ln \left( \frac{1}{V_y} \int_{\mathcal{Y}} \exp\left[ -E(\mathbf{x}; \mathbf{y}) / k_B T \right] dV_y \right)
$$

(6)

with

$$
V_y = \int_{\mathcal{Y}} dV_y .
$$

Expanding Eqn. (6) into a power series in $\beta = 1/k_B T$ (the so-called cumulant expansion [67]), we obtain Eqn. (7).

$$
F(\mathbf{x}) = U_1 - \frac{1}{2} (U_2 - U_1^2) \beta + \frac{1}{6} (U_3 - 3U_1 U_2 + 2U_1^3) \beta^2 - \frac{1}{24} (U_4 - 3U_2^2 - 4U_1 U_3 + 12U_1^2 U_2 - 6U_1^4) \beta^3 + \cdots
$$

(7)

where

$$
U_k = \frac{1}{V_y} \int_{\mathcal{Y}} E(\mathbf{x}; \mathbf{y})^k dV_y
$$

(8)

is the $k$th moment of the energy about $E = 0$. 
The presence of the integrals of various powers of energy gives rise to the appearance of multibody terms in the average energy of the simplified system. Consider for example, three peptide groups that are in a close contact with each other, without the presence of the solvent. The forces acting between the individual atoms of the peptide groups are, to a good approximation, pairwise. If we consider only the peptide-group centers and average the interaction energy over the internal degrees of freedom of each peptide group according to Eqn. (7), \( U_1 \) will still include only the interactions of the pairs of the centers. We have shown elsewhere [59, 61] that in this specific case \( U_1 \) happens to be zero, to a very good approximation. The integrals contained in the second moment of the energy, \( U_2 \), will already contain products of the interactions between two pairs of peptide groups, which in our example will share a common peptide group, which could create three-body contributions to the average energy. However, in the specific case of the peptide groups, the three-body contributions to \( U_2 \) happen to vanish, to a very good approximation [59, 61]. Nevertheless, in the case of \( U_3 \) and higher energy moments we cannot avoid considering the terms that depend on the coordinates of more than two centers, which creates multibody contributions to the energy.

Based on the above general considerations and taking advantage of our dipole model of the peptide group, we have derived analytical expressions for the multibody terms in of peptide-group interactions [63]. We found that the cooperative contribution to the averaged energy is significant, if some of the peptide groups are adjacent in the chain. A frequent case is the cooperativity of two pairs of adjacent peptide groups; they are found in \( \alpha \)-helices and \( \beta \)-sheets. In this case \( U_{\text{corr}} \) is expressed by Eqn. (9).

\[
U_{\text{corr}}(i, i - 1; k, k \pm 1) = \\
= -z^2 \left[ 9\eta_{i,i-1,k,k\pm1} + \\
+ 4\eta_{i,i-1,k\pm1} \right]
\]

where \( \eta_{ik} \) is the energy of the interaction of two aligned peptide-group dipoles (or two hydrogen-bonded peptide groups), while \( \eta_{ik} \) is the interaction of two antiparallel peptide-group dipoles (or two antiparallelly stacked peptide groups); \( z^2 \) is a proportionality constant. This is illustrated in Fig. 4. It should also be noted that these four-body terms have very much in common with the terms introduced by Skolnick and coworkers on a heuristic basis [37, 38]. For the derivation and the functional forms of the multibody terms the reader is referred to the original paper* [63].

**PARAMETERIZATION OF THE FORCE FIELD**

The following procedures are commonly used to parameterize united-residue potentials:

1. Direct averaging [Eqn. (5)] of the all-atom potentials over the “less important” degrees of freedom that are lost when passing from the all-atom to the united residue representation of the polypeptide chain [26–29].

2. Determination of the united-residue potentials so as to reproduce the single body, pair, and possibly triplet distribution functions, as well as contact free energies determined from protein crystal data [47–49, 54–56]. This approach is based on the following assumptions:

   (a) The distribution functions obtained by using a sufficiently large number of protein crystal data (each of which corresponds to a system at a free-energy minimum) are sufficiently good approximations to those of a hypothetical “stochastic” mixture. This approximation is justified by the observation that, although a crystal structure is at equilibrium as the whole structure, its individual parts can be forced to assume geometries far from locally equilibrated, locally lower-energy conformations having, however, higher probability of occurrence in the whole structure [68]. For example, the distributions of X–H bond lengths obtained from large data bases of crystal structures are qualitatively

*The paper is available at http://chemik.chem.univ.gda.pl:8000/ECCC3/19/poster.html
similar to those calculated from potential-energy surfaces of proton transfer [68].

(b) Interactions can be described with sufficient accuracy by using the potential of mean force, $W(\mathbf{X})$, $\mathbf{X}$ denoting the degrees of freedom of the considered subsystem, which are related directly to the corresponding distribution functions, $\rho(\mathbf{X})$:

$$\rho(\mathbf{X}) = \rho_0(\mathbf{X}) \exp[-\beta W(\mathbf{X})]$$  \hspace{1cm} (10)$$

where $\rho_0(\mathbf{X})$ is a known reference distribution function (e.g. the distribution function of non-interacting side chains tethered to the backbone).

3. A combination of the two preceding approaches in which some part of the potential is determined by direct averaging of the all-atom potential, e.g. the local and hydrogen-bonding interactions, and some estimated from protein crystal data. Such a division is motivated by the fact that, if direct averaging is computationally feasible, as in the case of the local and hydrogen-bonding interactions, the resulting potential will always be more accurate than that calculated from experimental distribution functions, whose accu-
racy is severely limited by the sparse number of protein crystal data. Conversely, obtaining the hydrophobic potential by direct averaging is in most cases not feasible, owing to the large number of degrees of freedom over which averaging must be carried out (i.e. the dihedral angles $\chi$ for each side chain) and possibly to the necessity of including explicit water molecules in the averaging.

4. Determination of the parameters of the potential so as to locate the native structures as global minima for a set of training proteins and, simultaneously, introducing a large energy gap between the near-native and non-native structures. For on-lattice simulations, such an approach based on spin-glass theory was developed by Wolynes and co-workers [9] and by Hao & Scheraga [10, 11]. This can be formulated in terms of the optimization of the so-called Z-score (Eqn. (11)):

$$Z = \frac{E_0 - 1 / N \sum_{i=1}^{N} E_i}{\sqrt{\left[\frac{1}{N} \sum_{i=1}^{N} E_i^2 - 1 / N^2 \left(\sum_{i=1}^{N} E_i\right)^2\right]}}$$

where $N$ is the number of conformations, $E_0$ is the energy of the native conformation, and $E_i$ is the energy of the $i$-th non-native conformation.

The value of the Z-score is the normalized difference between the energy of the native conformation and the mean energy of the quasi-continuous energy distribution corresponding to non-native structures. The more negative the Z-score values, the more the native structure is distinguished from non-native ones. A similar method was developed for off-lattice simulations by Crippen and co-workers [50, 51].

Both in the first [59, 60] and in the second [61-63] generation of the force field we have implemented procedure 3 to determine the parameters of individual energy terms (the $U$'s). In the first version, the geometric and interaction parameters of the side chains were obtained based on the work of Levitt [26] and Miyazawa & Jernigan [31]. As mentioned in the preceding section, the peptide-group interaction potential, $U_{p_i p_j}$, was developed by averaging the simplified expression for the interaction energy of the peptide groups over the angles of the rotation of the peptide groups about the virtual-bond axis $C^\alpha-C^\alpha$ [59, 60]. This energy function, as well as the virtual-bond torsional energy, was parameterized through averaging of the all-atom ECEPP/2 [69, 70] potential [59, 60]. Because rigid virtual-valence geometry was assumed, the terms $U_1$ and $U_{ro}$ were absent. The correlation term, $U_{c_1}$ was not considered either. Finally, the relative weights of the energy terms were estimated so as to achieve compatibility of some energies estimated from different parameterization procedures (e.g. the contact energy of the interaction of two glycine residues calculated from the PDB by Miyazawa & Jernigan [31] can be considered as the energy of the interaction of two peptide groups and should therefore be equal to the average energy of peptide-group interaction obtained by averaging the ECEPP/2 all-atom potential [60, 61]).

In the second generation of the force field, the side-chain [61] and local-interaction [62] terms were parameterized based on pair-correlation and distribution functions calculated from 195 non-homologous high-resolution protein structures from the PDB. The electrostatic term ($U_{p_i p_j}$) was inherited in unchanged form from the preceding version of the force field. Further addition was the determination of the weights of the energy terms so as to optimize the Z-score of the phosphocarrier protein from Streptococcus faecalis (1PTF; 87 amino-acid residues; 1.6 Å resolution) by means of inverse-folding calculations [62]. Its structure is shown in Fig. 5. The procedure of weight determination was as follows [62]:

1. A database of contiguous $C^\alpha$ traces was selected from the PDB (a total of 502).
2. 1PTF sequence was superposed on a series of randomly chosen $C^\alpha$ traces (about 500) and on the native pattern of 1PTF.
3. For each pattern, the geometry was regularized by performing a series of energy minimizations with $C^\alpha$ distance constraints with gradually diminished weights of the distance constraints.
4. The energy of the virtual chain was minimized with no distance constraints and the resulting energy was considered
Figure 5. The structures of the proteins used in inverse-folding calculations: histidine-containing phosphocarrier protein (1PTF), calcium-binding protein (4ICB), ubiquitin (1UBQ), α-spectrin (1SHG), major cold-shock protein (1MJC), and cytochrome b5c (3B5C).
Table 1. Iterative determination of self-consistent weights of energy terms for 1PTF (data from [62])

<table>
<thead>
<tr>
<th>Iter.</th>
<th>$\omega_e^{0a}$</th>
<th>$\omega_f^{0a}$</th>
<th>$\omega_l^{0a}$</th>
<th>$\Delta E_{nat}^b$</th>
<th>$Z$</th>
<th>$\omega_e^{*a}$</th>
<th>$\omega_f^{*a}$</th>
<th>$\omega_l^{*a}$</th>
<th>$\Delta E_{nat}^b$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.458</td>
<td>0.638</td>
<td>1.692</td>
<td>+20.0</td>
<td>-1.68</td>
<td>0.249</td>
<td>0.050</td>
<td>0.271</td>
<td>-25.8</td>
<td>-4.67</td>
</tr>
<tr>
<td>1</td>
<td>0.610</td>
<td>0.186</td>
<td>0.846</td>
<td>-16.8</td>
<td>-4.26</td>
<td>0.341</td>
<td>0.112</td>
<td>0.403</td>
<td>-21.0</td>
<td>-4.96</td>
</tr>
<tr>
<td>2</td>
<td>0.341</td>
<td>0.117</td>
<td>0.403</td>
<td>-23.5</td>
<td>-4.41</td>
<td>0.650</td>
<td>0.190</td>
<td>0.600</td>
<td>-26.4</td>
<td>-4.74</td>
</tr>
<tr>
<td>3</td>
<td>0.495</td>
<td>0.150</td>
<td>0.201</td>
<td>-14.4</td>
<td>-3.80</td>
<td>0.347</td>
<td>0.201</td>
<td>0.091</td>
<td>-12.7</td>
<td>-3.87</td>
</tr>
<tr>
<td>4</td>
<td>0.421</td>
<td>0.175</td>
<td>0.147</td>
<td>-13.4</td>
<td>-3.78</td>
<td>0.444</td>
<td>0.144</td>
<td>0.042</td>
<td>-14.0</td>
<td>-3.80</td>
</tr>
</tbody>
</table>

*$\omega^a$, $\mu^a$ denote the initial weights used to carry out the threading-with-minimization calculations of a given iteration and the final weights optimized in this iteration, respectively. In order to avoid oscillations, the initial weights of the next iteration were arithmetic means of the weights optimized in two preceding iterations. $\Delta E_{nat}^b = E_{nat} - E_{min}$ [62], where the latter term is the minimal element in the set of energies of all non-native structures.

as the final energy corresponding to the pattern.

1. The $Z$-score was calculated (Eqn. (11)) and then minimized as a function of energy-term weights.

2. The procedure was iterated from step 2 with new weights, until the weights calculated in two consecutive iterations were reasonably consistent.

The history of the determination of energy-term weights is summarized in Table 1. As shown, although the initial weights did not produce an energy function localizing the native structure of 1PTF as the lowest-energy structures, a folding potential was obtained already in the first iteration.*

SEARCH OF THE CONFORMATIONAL SPACE OF THE SIMPLIFIED AND OF THE ALL-ATOM CHAIN

In order to search the conformational space of the simplified chain, the Monte Carlo with Minimization Method of Li & Scheraga [16, 17] was implemented. In brief, the method consists of the following steps:

1. Choose an arbitrary starting conformation.

2. Minimize the energy; let the geometric parameters of the resulting conformation be contained in the vector $\Gamma_0$ and the corresponding energy as $U_0$.

3. Perturb $\Gamma_0$ according to a predetermined scheme.

4. Carry out energy minimization, obtaining the conformation $\Gamma_1$ and energy $U_1$.

5. If neither $U_1$ nor $\Delta U_1$ differs by more than a preassigned cut-off from the previous conformations, discard it and repeat the process beginning at step 3; otherwise apply a Metropolis test [71] in order to accept or reject the conformation.

6. If the new conformation is accepted, substitute $\Gamma_1$ for $\Gamma_0$, and $U_1$ for $U_0$, and repeat from step 3.

7. Iterate steps 3–7, until the requested number of accepted conformations is obtained.

In order to search the conformational space of the chains with all-atom backbone, the more efficient Electrostatically Driven Monte Carlo Method (EDMC) [18] was applied. This method is a modified version of MCM, in which a part of perturbations (step 3) is done so as to align the worst oriented peptide groups in the electrostatic field of the remaining part of the polypeptide chain. All-atom calculations were carried out using the ECEPP/2 [69, 70, 72] and subsequently the ECEPP/3 [73] force field, supplemented with the SRFOPT (surface) model of hydration [74].

*The parameters of the second-generation force field are available at http://chemik.chem.univ.gda.pl:8000/local/docs/adam; see the file readme.txt.
Figure 6. Illustration of the conversion of the lowest-energy united-residue structure of the avian polypeptide (APP) to the all-atom structure: a) the Cα trace; b) Cα trace with indicated dipole paths; it should be noted that there are three paths, each one connecting every third peptide groups of the α-helical part running from residue 18 to 36; c) all-atom backbone constructed based on peptide-group dipole orientation followed by perturbation of the structure by means of the EDMC method [18] to find a relaxed structure; d) side chains attached with subsequent EDMC perturbation to give the all-atom structure.
RECONSTRUCTION OF THE ALL-ATOM CHAIN FROM THE SIMPLIFIED CHAIN

The method consists of two stages: i) positioning the peptide groups between the α-carbons given the α-carbon trace, which gives the complete all-atom backbone and ii) positioning the side chains for on the all-atom backbone [59, 60].

The approach used to solve problem i) was based on the observation that the peptide groups in protein structures tend to form an extensive hydrogen bond network, which can also be expressed in terms of their tendency to optimal electrostatic interactions [15]. Taking advantage of the dipole model of the peptide group [15, 59], the problem of positioning the peptide groups given the Cα trace can be formulated as the problem of optimal alignment of peptide-group dipoles. In regular helical or sheet structures, lines of aligned peptide-group dipoles or dipole paths can be distinguished, which comprise all peptide groups in regular structures (e.g., each α-helix contains three paths linking every third peptide group). We therefore proposed the following algorithm for positioning of the peptide groups [59], which we named the dipole path method:

1. Find the chains of non-contiguous peptide groups, such that the average energy of the electrostatic interactions of the neighboring peptide groups is below a chosen cut-off limit and the peptide groups lie on a line with a low curvature.
2. Align the peptide-group dipoles along each path. Because alignment can be carried out in two directions, the correct direction is chosen taking into account the electrostatic interactions of the dipoles of the neighboring paths and the local interactions within the amino-acid residues involved in the dipole paths.
3. Align the “isolated” peptide groups in the electrostatic field of the determined dipole paths.
4. Based on the position of peptide-group dipoles, calculate the coordinates of all atoms in the peptide groups. The side chains are still represented by single interaction sites.

5. Complete the determination of backbone geometry by carrying out short MCM runs (or even just a single energy minimization) with Cα distance constraints corresponding to the parent united-residue structure.

Once the all-atom backbone has been obtained, structure generation is completed by putting on all-atom side chains in such a manner that collisions between the atoms are avoided and carrying out EDMC simulations with and subsequently without the Cα distance constraints [60]. The process of all-atom chain building is illustrated in Fig. 6

RESULTS OBTAINED WITH THE FIRST-GENERATION FORCE FIELD

The first test of the united-residue force field and the whole protocol for protein structure prediction was the avian pancreatic polypeptide (APP)—a small 36-residue protein. Its crystal structure was determined by Blundell et al. [75] at 1.4 Å and then by Glover et al. [76] at the 0.98 Å resolution. The structure consists of a polyproline-like or collagen-like helix running from residue 1 to 8, packed against the hydrophobic face of an α-helix which extends from residue 13 to 31. The C terminus does not participate in the α-helix. Although the molecule forms a dimer both in solution [77, 78] and in the crystal phase [75, 76], there is some evidence that even the monomer is sufficiently stabilized by hydrophobic interactions between the two domains to retain the X-ray conformation [77, 78, 75].

The lowest-energy structure of APP obtained with the united-residue force field had an r.m.s. deviation from the Cα-trace of the 1PPT structure of 3.8 Å [60]. After applying the conversion procedure, the r.m.s. deviation increased to 4.1 Å; however the structure still resembled the native structure (see Fig. 7 for superposition of both structures). This means that the quality of the united-residue force field is critical for the success of the procedure; perturbing the converted structure using the all-atom force field does not bring it closer to the native structure. This feature of the hierarchical protocols for pro-
tein-structure prediction was also observed by Skelnick and coworkers [37, 38].

The procedure was subsequently applied in de novo prediction of the structure of the 29-residue neuropeptide galanin isolated from a number of mammal species [79]. No X-ray structure is available for this peptide. Our calculations had suggested that even in aqueous solution this peptide should contain a considerable fraction of helical structure [80]; this was later confirmed by NMR measurements [81].

The performance of the first-generation force field was, however, significantly poorer with more complicated structural motifs that included β-sheets, such as the epidermal growth factor (EGF), ferredoxin, and crambin, although it was able to distinguish the native-like structures from alternative structures as low-energy ones. The resulting best structures were, however, too much distorted with respect to the native structures and resembled molten globules rather than the native structures, though they essentially had the native-like packing of the side chains (Liwo, A., Pincus, M.R., Wawak, R.J., Rackovsky, S. & Scheraga, H.A., unpublished results). This could be caused both by the as-

Table 2. Summary of control threading experiments using all test proteins with weights determined using the phosphocarrier protein (1PTF); initial weights of iteration 4 of Table 1 (data from [92])

<table>
<thead>
<tr>
<th>Protein</th>
<th>N</th>
<th>Type</th>
<th>Cofactor</th>
<th>Initial</th>
<th>Optimized</th>
<th>r.m.s.</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>∆E_{nat}</td>
<td>Z-score</td>
<td>∆E_{nat}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(kcal/mol)</td>
<td></td>
<td>(kcal/mol)</td>
</tr>
<tr>
<td>4ICB</td>
<td>76</td>
<td>α</td>
<td>Ca^{2+}</td>
<td>-24.6</td>
<td>-4.84</td>
<td>-27.6</td>
</tr>
<tr>
<td>1UBQ</td>
<td>76</td>
<td>β + α</td>
<td>none</td>
<td>-15.5</td>
<td>-3.29</td>
<td>-10.1</td>
</tr>
<tr>
<td>3B5C</td>
<td>85</td>
<td>α + β</td>
<td>heme, Fe^{2+}</td>
<td>-13.5</td>
<td>-3.66</td>
<td>-14.8</td>
</tr>
<tr>
<td>1SHG</td>
<td>57</td>
<td>β</td>
<td>none</td>
<td>-5.2</td>
<td>-3.06</td>
<td>-5.2</td>
</tr>
<tr>
<td>1MJJC</td>
<td>69</td>
<td>β</td>
<td>none</td>
<td>-3.9</td>
<td>-3.40</td>
<td>-7.8</td>
</tr>
</tbody>
</table>

*See Fig. 5 for the names of these proteins. *The number of amino-acid residues. *Values calculated using the final weights obtained for the phosphocarrier protein (1PTF) (ω^4 of iteration 4 from Table 1). *Values calculated using the weights optimized using the energies obtained in threading-with-minimization calculations for a given protein. *r.m.s. deviation from the native structure.
sumption of rigid virtual-valence geometry and the absence of multibody interactions in the force field.

PRELIMINARY RESULTS OBTAINED WITH THE SECOND-GENERATION FORCE FIELD

Using the weights determined from the inverse-folding calculations on the phosphocarrier protein (IPTF), we checked the ability of the potential to locate the native structures of other proteins correctly, using the inverse-folding approach [62]. In these calculations, the force field did not include the correlation term $U_{cor}$. Table 2 summarizes the results of these calculations for a number of monomeric proteins of length exceeding 50 amino-acid residues and the structures are shown in Fig. 5. As shown, in each case the native structure is the lowest in energy and is separated from non-native structures by a significant energy gap. The Table also includes weights optimized specifically for each protein after the appropriate series of threading with-minimization calculations (cf. Section "Parameterization of the force field"). As shown, optimization of weights in this manner did not result in major decreases of the $Z$-score values or the energy differences between the native and lowest-energy non-native conformation, which means that weights obtained by determining the weights of the energy terms using the phosphocarrier protein (IPTF) are also relevant for other proteins. It should be noted that none of the above proteins was used in parameterization of the potential.

Use of energy minimization in our threading calculations gives a possibility that the procedure will find a structure close to the native pattern of the target protein, even if the structural fragments from the data base are distant from its native structure. Table 3 summarizes the results of threading-with-minimization calculations for the 10–58 fragment of the B-domain of staphylococcal protein A. The native pattern of protein A was not present in the data base. As shown, all but one of the five lowest-energy patterns found in the data base are close to the native structure of protein A. Structures 4 and 5 were moved by energy minimization very far apart from the starting PDB structure, but they had become close to the native structure of protein A.

DIRECTIONS OF FURTHER WORK

Although the energy-term weights determined using the inverse-folding calculations of IPTF proved to be able to locate the native structures of other proteins as the lowest-energy ones, it is clear that for de novo folding this is insufficient to obtain reliable energy-term weights. The structures from the PDB

<table>
<thead>
<tr>
<th>Table 3. Summary of the results of the calculations on the 10–58 fragment of staphylococcal protein A (data from [63])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
</tr>
<tr>
<td>1BAB:D</td>
</tr>
<tr>
<td>1CSC</td>
</tr>
<tr>
<td>1ECA</td>
</tr>
<tr>
<td>2HMZ:C</td>
</tr>
<tr>
<td>1CPC:B</td>
</tr>
<tr>
<td>Native</td>
</tr>
</tbody>
</table>

*a* 4-digit code of the PDB entry followed by the chain code, if applicable; *b* the residue of the PDB structure onto which the first residue of protein A was superposed; *c* r.m.s. deviation of the energy-minimized structure from the original PDB pattern; *d* r.m.s. deviation from the NMR structure of protein A; *e* percentage of native contacts.
usually correspond to favorable local interactions and also consist of regular patterns. It can therefore be supposed that while they can serve to determine the relative weights of the hydrophobic and electrostatic term, the weights of the local and correlation term will be estimated poorly. At this moment, we are working on extending the determination of the energy-term weights (Eqn. 1) on the structures outside the PDB, applying the entropic-sampling [82] and other efficient methods to search the energy space. For the force field to be used efficiently in de novo folding, an efficient method of global optimization must be applied. We are now working on the application of the combination of the methods of deformation of the energy surface, such as the diffusion-equation and the shift method [19, 20, 24] with Monte Carlo methods. Such an approach has resulted in an efficient methods for finding the global energy minima of crystal structures of small molecules [83].

REFERENCES


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